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CONSTRAINTS ON THE LONG-RANGE PROPERTIES OF GRAVITY FROM WEAK GRAVITATIONAL LENSING

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ABSTRACT

Weak gravitational lensing provides a means of testing the long-range properties of gravity. Current measurements are consistent with standard Newtonian gravity and inconsistent with substantial modifications on Mpc scales. The data allows long range gravity to deviate from a $1/r$ potential only on scales where standard cosmology would use normal gravity but be dominated by dark matter. Thus, abnormal gravity theories must introduce two fine-tuning scales – an inner scale to explain flat rotation curves and an outer scale to force a return to Newtonian gravity on large scales – and these scales must coincidently match the scales produced by the dark matter theory after evolving the universe for 10 billion years starting from initial conditions which are exquisitely determined from the cosmic microwave background.

Subject headings: cosmology:theory – gravitational lensing

1. INTRODUCTION

Weak lensing of background galaxies by foreground large-scale structure offers an opportunity to directly probe the mass distribution on large scales over a wide range of redshifts. As first pointed out by Blandford et al. (1991) and Miralda-Escude (1991), these effects are of order a few percent in adiabatic cold dark matter models making their observation challenging but feasible. Early predictions for the power spectrum of the shear and convergence were made by Kaiser (1992) on the basis of linear perturbation theory. Jain & Seljak (1997) estimated the effect of non-linearities in the density through analytic fitting formulae (Peacock & Dodds 1996) and showed they substantially increase the power in the convergence below the degree scale. Because weak lensing can measure the matter power spectrum without many of the problems of approaches based on the distributions of galaxies or clusters (e.g. bias), it may ultimately provide as clean a cosmological probe as the microwave background. Recently, several observational groups have reported convincing evidence of the effect (van Waerbeke et al. 2000; Bacon et al. 2000; Kaiser et al. 2000; Wittman et al. 2000; Maoli et al. 2001; Rhodes et al. 2001; van Waerbeke et al. 2001)

All these theoretical and observational studies are primarily motivated by standard theories of gravity and cosmology. Despite the tremendous overall success of these theories, there has been a recent resurgence of interest in non-standard theories of gravity, largely motivated by the possibility that the standard paradigm has difficulty matching the dynamical structure of galaxies (e.g. Flores & Primack 1994; Moore 1994; Navarro & Steinmetz 2000). Most of these proposed modifications aim to make gravity a longer-ranged force on scales comparable to the sizes of galaxies in order to explain the flat rotation curves of galaxies on scales larger than the apparent distribution of matter (e.g. Sellwood & Kosowsky 2000, Sanders 1998, 1999, 2000; McGaugh 1999, 2000, but see Scott et al. 2001 and Aguirre et al. 2001 for an opposite perspective). As has been noted before (e.g. Krisher 1988; Walker 1994; Bekenstein & Sanders 1994; Zhytnikov & Nester 1994; Edery 1999; Kinney & Brisudova 2001; Uzan & Bernardeau 2001; Mortlock & Turner 2001) any longer ranged gravitational force, if it also affects photons, should have implications for gravitational lensing. In particular it should profoundly affect the strength of weak lensing shears on large scales. Many of the above authors, however, consider gravitational lensing by isolated objects. To understand the lensing effects of modifying gravity on large scales it is necessary to use the weak lensing formalism, summing over the contributions from all density perturbations.

2. THE MODEL

We base our models on the discussion by Zhytnikov & Nester (1994) of modified gravity theories within the context of linearized relativity (see also Edery 1999). This framework provides a relativistic gravity model which automatically obeys the equivalence principle and within which definite calculations can be made, while at the same time being as unrestrictive as possible. Further discussion of the experimental foundations for the assumptions can be found in Zhytnikov & Nester (1994) and in Weinberg (1972), Misner, Thorne & Wheeler (1973) and especially Will (1993, §§2-3).

For any such model, the important change in the formalism for the propagation of light through such a weak field metric is to change the Poisson equation relating the density to the potential whose derivative is used to determine the bend angle of photons. The angular power spectrum of the convergence, κ, can be written as an integral over the line-of-sight of the power spectrum of the density fluctuations (Kaiser 1992). For sources at a distance D_s,

$$\ell(\ell+1)C_\ell/(2\pi) = \frac{9\pi}{4\ell}\left[\Omega_m H_0^2 D_s^2\right]^2 \int \frac{dD}{D_s} t^3(1-t)^2 \times \left[\frac{\Delta^2_{\rm mass}(k=\ell/D,a)}{a^2}\right] f^2(k=\ell/D), \quad (1)$$

where $t \equiv D/D_s$, $\Delta^2_{\rm mass}(k) = k^3P(k)/(2\pi^2)$ is the contribution to the mass variance per logarithmic interval physical wavenumber and $\ell(\ell+1)C_\ell/(2\pi)$ is the contribution to $\kappa^2_{\rm rms}$ per logarithmic interval in angular wavenumber (or equivalently multipole) ℓ. The only change from the standard result is that the Poisson equation relating the potential to the density perturbations is modified from $f(k) = 1$ to a functional form determined by the Poisson equation of the modified theory of gravity. On small physical scales (large wavenumber k), $f(k) = 1$ is required to be consistent with the known properties of gravity.

If the sources have a range of redshifts then one simply integrates the above expression over the redshift distribution of the sources. We shall assume throughout that

$$\frac{dn}{dD} \propto D \exp[-(D/D_*)^4] \tag{2}$$

and fix D_* by the requirement that $\langle z_{\rm src} \rangle = 1$. In evaluating Eq. (1), we will use the method of Peacock & Dodds (1996) to compute the non-linear power spectrum as a function of scale-factor. Throughout we shall use the concordance cosmology of Ostriker & Steinhardt (1995) since it provides a reasonable fit to recent CMB, weak lensing and large-scale structure data. For this choice of parameters the lensing kernel peaks at $z \simeq 0.43$ at a (comoving) angular diameter distance of $1150h^{-1}$ Mpc.

In our calculation we only consider the propagation of rays through a known density distribution, and we model that known density distribution using a standard cosmological model viewed as a means to interpolate the evolution of structure with redshift. We do not attempt to self-consistently form the observed structures using the modified gravitational potential[1]. If we assume that all theories must match the local density distribution, the only consequence of this assumption is that the evolution of structure implicit in Eq. (1) uses the standard growth rates rather than those of the modified gravity.

[1]In the model described below, a linear fluctuation analysis suggests that long-wavelength modes would grow more slowly than the standard model would predict. Thus neglect of this effect is conservative if we start from an initially scale-invariant spectrum.

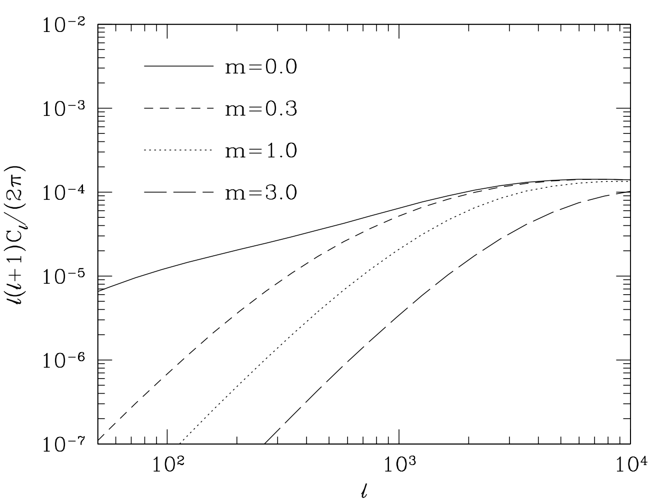

FIG. 1.— The angular power spectrum, $\ell(\ell+1)C_\ell/(2\pi)$, vs. multipole moment ℓ for models with $\alpha = 1.0$ and $m = 0$, 0.3, 1.0 and $3.0h\,{\rm Mpc}^{-1}$. The sources are assumed to have $\langle z_{\rm src} \rangle = 1$. Spectra for other values of α can be roughly obtained by averaging the $m = 0$ spectrum and the appropriate $\alpha = 1$ spectrum (plotted here) with the relevant weights.

Examining the effects of modified gravity simply becomes a question of considering different structures for the function $f(k)$. In 4D, the metric, being symmetric, contains 10 functions. The 4 constraints of energy-momentum conservation reduce the number of free functions to 6. These 6 free functions can be decomposed under rotations as 2 scalar (density perturbations), 2 vector (vortical motions) and 2 tensor (gravity wave) modes. Within the linearized theory there are a number of propagating modes, which have the form of Yukawa (exponential) potentials

$$U(\vec{r};m) = G \int \frac{\rho(\vec{r'})d^3r'}{|\vec{r}-\vec{r'}|} e^{-m|\vec{r}-\vec{r'}|} \quad . \tag{3}$$

Under a variety of reasonable assumptions Zhytnikov & Nester (1994) conclude that the most general metric describes forces mediated by massive and massless scalar and tensor particles. We follow Zhytnikov & Nester (1994) in neglecting the vector modes, however we will allow arbitrary couplings for the scalar and tensor modes. In general relativity in the weak field limit

$$g_{00} = (-1 + 2U) \tag{4}$$
$$g_{ij} = (\ \ 1 + 2U)\,\delta_{ij} \tag{5}$$

where U is the usual Newtonian potential. The metric of Zhytnikov & Nester (1994) has the same form, but with Yukawa potentials in addition to the Newtonian one.

For test particles with $v \ll c$ or fluids with $p \ll \rho c^2$ only the time-time part of the metric is relevant, the contribution of the g_{ij} terms being suppressed by $\mathcal{O}(v^2/c^2)$. However, for light, the bend angle due to the potential is actually the arithmetic mean of the coefficients in g_{00} and g_{ij}. Though the extra scalar and tensor modes can enter into the space-space and time-time part of the metric differently, we shall consider the 1 parameter family of models where these coefficients are equal. As Kinney & Brisudova (2001) discuss, the requirement that cluster mass estimates from galaxy dynamics, pressure equilibrium of the X-ray gas and gravitational lensing agree means that any modified gravity law must affect photon propagation in roughly the same was as it affects particle orbits. A modified gravity which differentially affects particles and photons will almost always lead to a discrepancy between these three cluster mass estimates.

Thus in our model, in the weak field limit, the propagation of light is the same as in standard general relativity, except that the potential is

$$U(\vec{r}) = (1-\alpha)U(\vec{r},0) + \alpha U(\vec{r},m) + \cdots \tag{6}$$

where $\cdots$ represents possible other terms of the same form as the second. We shall further simplify our calculation by considering only 1 correction term in what follows. In such a theory with one additional "field", the function appearing in the estimate of the weak lensing power spectrum is

$$f(k) = (1-\alpha) + \alpha\, \frac{k^2}{k^2+m^2}, \tag{7}$$

where $\alpha = 0$ for standard gravity and $\alpha \simeq -0.9$ and $m^{-1} \sim 50\,$kpc in order to produce flat rotation curves without dark matter (e.g. Sanders 1986). The corresponding potential for an object of mass M simplifies to the Newtonian result, $-GM/r$, on small scales where $mr \ll 1$, and has a different effective coupling constant, $-GM(1-\alpha)/r$ on large scales, $mr \gg 1$.

Fig. 1 shows the anisotropy spectrum predicted for a range of models. If we limit the range of gravity ($\alpha > 0$) then the shear fluctuations on large angular scales are suppressed, and if we extend the range they are enhanced. This should be a generic feature of any modification to the long-range force law. To obtain limits on the parameters in our model we calculated the rms shear expected in Gaussian windows with FWHM of $5'$ and $10'$ as a function of α and m (Fig. 2). These predictions are consistent with the rms shear measured on these scales by van Waerbeke et al. (2001) only for models with parameters close to those of standard gravity. We can minimize the model dependence of the result by examining the ratio of the power at $5'$ and $10'$, as this largely removes any dependence of the result on the matter density and the normalization of the power spectrum. In Fig. 3, we see that the data are consistent with standard gravity and a broad range of alternate theories. These theories are acceptable because our alternate gravity model has a $1/r$ potential on large scales so that when the $5'$ scale corresponds to a physical scale larger than m^{-1}, the change in the coupling constant α is degenerate with a change in the enclosed mass. For sources with a mean redshift of unity, the $5'$ scale corresponds to a length scale at the peak of the lensing kernel of approximately $1h^{-1}$ Mpc.

Theories which do not return to a $1/r$ potential on large scales are relatively easy to rule out (see Walker 1994). Assuming that the bend angle of light remains proportional to the gradient of the projected gravitational potential, such theories predict that random lines of sight would be

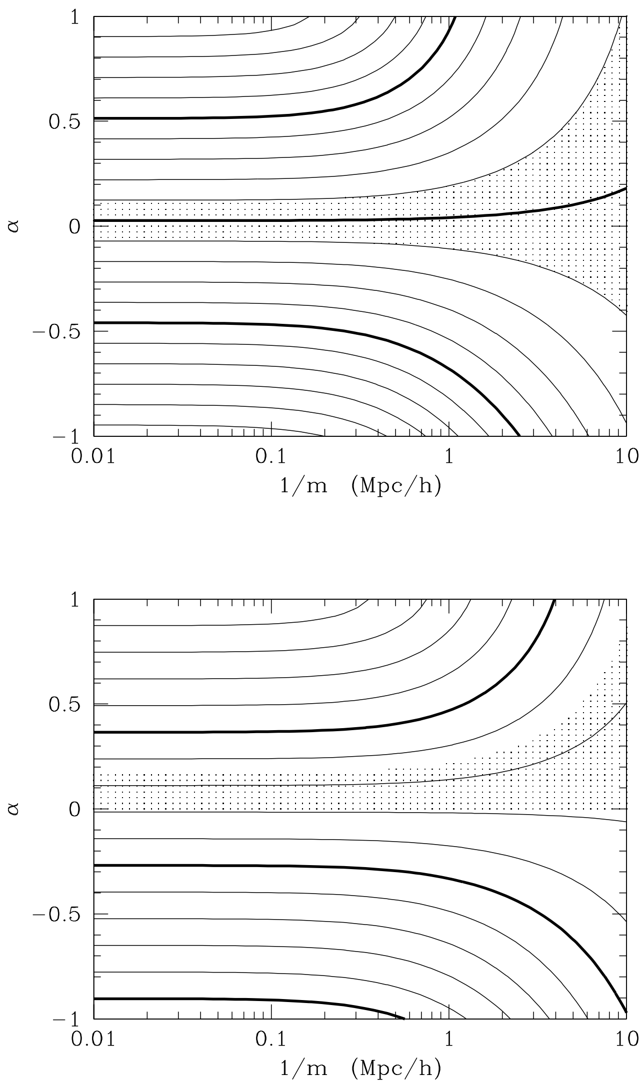

Fig. 2.— The rms shear, smoothed with a $5'$ FWHM gaussian (top) or a $10'$ FWHM gaussian (bottom), predicted for the "concordance" cosmology with $\Omega_{\rm mat} = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.67$ and $\sigma_8 = 0.9$ as a function of α and m (h/Mpc). Contours are spaced every 0.001 with bold contours indicating 0.005 (top), 0.01 and 0.015 (bottom). The stippled regions are consistent (at 1σ) with the van Waerbeke et al. (2001) measurements.

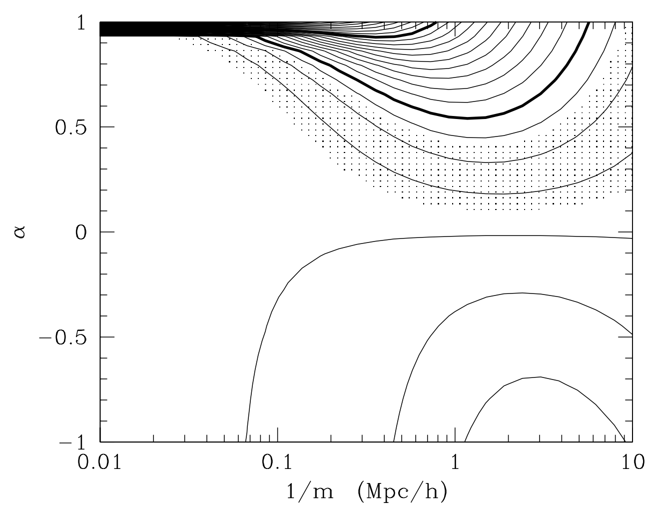

Fig. 3.— The ratio of the rms shear on $5'$ and $10'$ scales for the same cosmology as the previous figure. Contours are spaced in steps of 0.05, increasing to top left. Thick contours are spaced every 0.5, starting at 1.5. The stippled region is consistent (at 1σ) with the van Waerbeke et al. (2001) measurements.

highly sheared and (de)magnified[2] in contradiction with observations. This problem can be traced to the lack of degeneracy between renormalizing the mass and adjusting the coupling constant. For example, ignoring the Kinney & Brisudova (2001) *ansatz* for permissible forms of alternate gravity, we could use the force law

$$-\phi'(r)/GM = -\frac{1}{r^2} - \frac{\exp(-mr)}{rr_0} \quad (8)$$

which is Keplerian for $r \ll r_0$ and $r \gg m^{-1}$ but is a $1/r$ force law, producing a flat rotation curve, in between. The potential corresponding to this force law is

$$\phi/GM = -1/r + \mathrm{Ei}[-mr] \quad (9)$$

where $\mathrm{Ei}[x]$ is the exponential integral. The corresponding kernel for the weak lensing integral is

$$f(k) = 1 - \frac{km - (k^2+m^2)\tan^{-1}(k/m)}{r_0 k(k^2+m^2)}. \quad (10)$$

Figure 4 shows the angular power spectrum in this model for a range of scales r_0 and a large outer cutoff $m^{-1} = 50h^{-1}$ Mpc. Compared to normal gravity, the modified theories have enormously enhanced large scale power and very different shapes.

3. DISCUSSION

Current modified gravity theories tuned to explain the rotation curves of galaxies work in a standard cosmology because we measure rotation curves only where there are baryons. We can see that the rotation curve is flat out to the limit where there are no more baryons to measure, but we cannot see that it is Keplerian as we approach the edge of the more extended dark matter distribution. If we could continue to trace rotation curves on larger scales we would see a growing difference between standard cosmological models and theories using modified gravitational physics.

Weak lensing allows us to do this experiment, although on such large scales we must sum over the contributions of all of the mass rather than consider the rotation curves of discreet objects. As we would expect qualitatively, increasing the strength of the gravitational field at long ranges predicts stronger weak lensing signals on large scales than standard cosmological models. Current measurements of the rms shear on scales of $5'$-$10'$ rule out the theories we consider in the parameter ranges where they could explain rotation curves without dark matter unless the deviation from normal gravity is limited to a restricted range of spatial scales from $10h^{-1}\,\mathrm{kpc} \lesssim r \lesssim 1h^{-1}\,\mathrm{Mpc}$. On larger scales the models must return to the r^{-2} force law of normal gravity in order to be consistent with measurements.

In standard cosmological models, once we postulate the existence of dark matter, the inner and outer scales appear naturally. On small scales the cooling of the baryons concentrates the baryons relative to the dark matter and renders them luminous and detectable. Thus, normal matter combined with normal gravity naturally explain dynamics on scales $\lesssim 10h^{-1}$ kpc. On intermediate scales, dark matter provides an additional source of density, which can be interpreted as an abnormal gravitational theory using only the visible baryons as sources. On large scales the universe returns to homogeneity, and the special properties of the $1/r^2$ force law make the weak lensing power slowly diminish on large scales. Abnormal, longer ranged theories lose the cancellation properties of the $1/r^2$ force law on large scales, despite the increasing homogeneity of the density on these scales, leading to enormous enhancements in the strength of the weak lensing shear. Such strong shears are in gross disagreement with even the first generation of weak lensing measurements on these scales (van Waerbeke et al. 2000; Bacon et al. 2000; Kaiser et al. 2000; Wittman et al. 2000; Maoli et al. 2001; Rhodes et al. 2001; van Waerbeke et al. 2001). Thus, abnormal gravity theories must introduce two fine-tuning scales – an inner scale to explain flat rotation curves and an outer scale to force a return to Newtonian gravity on large scales – and these scales must coincidently match the scales produced by the dark matter theory after evolving the universe for 10 billion years starting from initial conditions which are exquisitely determined from the cosmic microwave background.

Finally, although we lack a formalism for estimating weak lensing in non-potential theories such as MOND, Mortlock & Turner (2001) have emphasized that weak lensing results should be generic, as it requires only that photons and particles have similar responses to gravitational fields. This similarity of behavior is observed on the relevant scales (Mpc) through the near equivalence of weak lensing, dynamical, and X-ray determinations of cluster masses (Kinney & Brisudova 2001).

We would like to thank L.V. van Waerbeke for providing

[2]For example, for a $\log r$ potential and a Poisson distribution of lenses the convergence, κ, of a source at D_s (assumed to be much larger than the scale, r_0, beyond which gravity is $\log r$) is $\kappa \simeq \pi\alpha_0 \left(nr_0^2 D_s\right) \gg 1$ for any reasonable source density n.

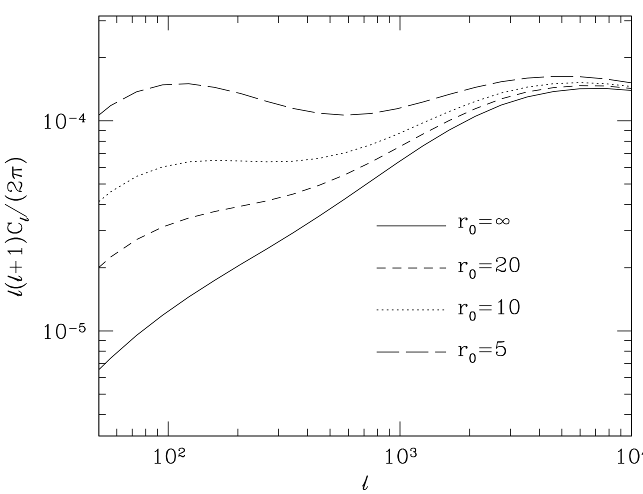

FIG. 4.— The angular power spectrum, $\ell(\ell+1)C_\ell/(2\pi)$, vs. multipole moment ℓ for our second model with $m^{-1} = 50h^{-1}$ Mpc and $r_0 = \infty$, 20, 10 and $5h^{-1}$ Mpc. As $r_0 \to \infty$ the model becomes standard gravity. Note the change in shape and the enormous enhancement in the power on large scales.

more details of their VIRMOS survey results. M.W. was supported by NSF-9802362 and a Sloan Fellowship. C.S.K. was supported by the Smithsonian Institution and NASA grants NAG5-8831 and NAG5-9265.

REFERENCES

Aguirre A., Burgess C.P., Friedland A., Nolte D., 2001, preprint [hep-ph/0105083]
Bacon D., Refrigier A., Ellis R., 2000, MNRAS, 318, 625
Blandford R.D., Saust A.B., Brainerd T.G., Villumsen J.V., 1991, MNRAS, 251, 600
Bekenstein J.D., Sanders R.H., 1994, ApJ, 429, 480
Edery A., 1999, Phys. Rev. Lett., 83, 3990
Flores, R., & Primack, J.R., 1994, ApJ, 427, L1
Jain B., Seljak U., 1997, ApJ, 484, 560 [astro-ph/9611077]
Kaiser N., 1992, ApJ, 388, 272
Kaiser N., Wilson G., Lupino G., 2000, preprint [astro-ph/0003338]
Kinney W.H., Brisudova M., 2001, in "Proceedings of the 15th Florida Workshop in Nonlinear Astronomy and Physics" [astro-ph/0006453]
Krisher T.P., 1988, ApJ, 331, L135
Maoli R., et al., 2001, A&A, in press [astro-ph/0011251]
McGaugh S., 1999, ApJ, 523, L99 [astro-ph/9907409]
McGaugh S., 2000, ApJ, 541, L33 [astro-ph/0008188]
Miralda-Escude J., 1991, ApJ, 380, 1
Misner C.W., Thorne K.S., Wheeler J.A., 1973, "Gravitation", Freeman, New York.
Moore, B., 1994, Nature, 370, 629
Mortlock D.J., E.L. Turner, 2001, preprint [astro-ph/0103208]
Navarro, J.F., & Steinmetz, M., 2000, ApJ, 528, 607
Ostriker J., Steinhart P.J., 1995, Nature, 377, 600
Peacock J. A., Dodds S. J., 1996, MNRAS, 280, 19
Rhodes J. Refregier A., Groth E., 2001, preprint [astro-ph/0101213]
Sanders, R.H., 1986, MNRAS, 223, 539
Sanders R.H., 1998, MNRAS, 296, 1009
Sanders R.H., 1999, ApJ, 512, L23
Sanders R.H., 2000, astro-ph/0011439
Scott D., White M., Cohn J.D., Pierpaoli E., 2001, preprint [astro-ph/0104435]
Sellwood, J.A., & Kosowsky, A., 2000, in Gas & Galaxy Evolution, Hibbar, Rupen & van Gorkom, eds., astro-ph/0009074
Uzan J.-P., Bernardeau F., 2001, preprint [hep-ph/0012011]
van Waerbeke L.V., Bernardeau F., Mellier Y., 1999, A&A, 342, 15 [astro-ph/9807007]
van Waerbeke L.V., et al., 2000, A&A, 358, 30
van Waerbeke L.V., et al., 2001, A&A submitted, astro-ph/0101511
Walker M.A., 1994, ApJ, 430, 463
Weinberg S., 1972, "Gravitation and cosmology", Wiley, New York.
Will C.M., 1993, "Theory and experiment in gravitational physics", Revised edition, Cambridge University Press, New York.
Wittman D.M., et al., 2000, Nature, 405, 143
Zhytnikov V.V., Nester J.M., 1994, Phys. Rev. Lett., 73, 2950